OCR
Oxford Cambridge and RSA

Wednesday 25 May 2016 – Morning

AS GCE MATHEMATICS

4722/01 Core Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4722/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

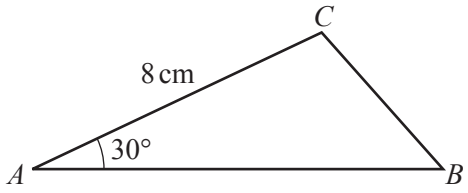
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1

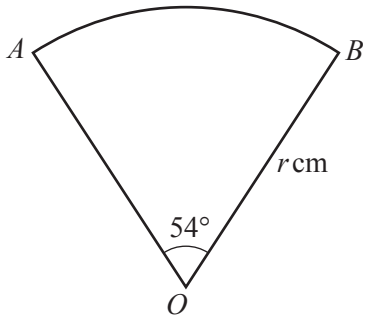


The diagram shows triangle ABC, with $AC = 8$ cm and angle $CAB = 30°$.

(i) Given that the area of the triangle is $20\,\text{cm}^2$, find the length of AB. **[2]**

(ii) Find the length of BC, giving your answer correct to 3 significant figures. **[2]**

2



The diagram shows a sector AOB of a circle with centre O and radius r cm. The angle AOB is 54°. The perimeter of the sector is 60 cm.

(i) Express 54° exactly in radians, simplifying your answer. **[2]**

(ii) Find the value of r, giving your answer correct to 3 significant figures. **[3]**

3 **(i)** Find the binomial expansion of $(3 + kx)^3$, simplifying the terms. **[4]**

(ii) It is given that, in the expansion of $(3 + kx)^3$, the coefficient of x^2 is equal to the constant term. Find the possible values of k, giving your answers in an exact form. **[2]**

4 **(i)** Express $2\log_3 x - \log_3(x + 4)$ as a single logarithm. **[2]**

(ii) Hence solve the equation $2\log_3 x - \log_3(x + 4) = 2$. **[4]**

5 **(a)** Find $\int (x^2+2)(2x-3)\,\mathrm{d}x$. **[3]**

(b) **(i)** Find, in terms of a, the value of $\int_1^a (6x^{-2}-4x^{-3})\,\mathrm{d}x$, where a is a constant greater than 1. **[4]**

(ii) Deduce the value of $\int_1^\infty (6x^{-2}-4x^{-3})\,\mathrm{d}x$. **[1]**

6 An arithmetic progression $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 5$ and $u_{n+1} = u_n + 1.5$ for $n \geqslant 1$.

(i) Given that $u_k = 140$, find the value of k. **[3]**

A geometric progression $w_1, w_2, w_3, \ldots$ is defined by $w_n = 120 \times (0.9)^{n-1}$ for $n \geqslant 1$.

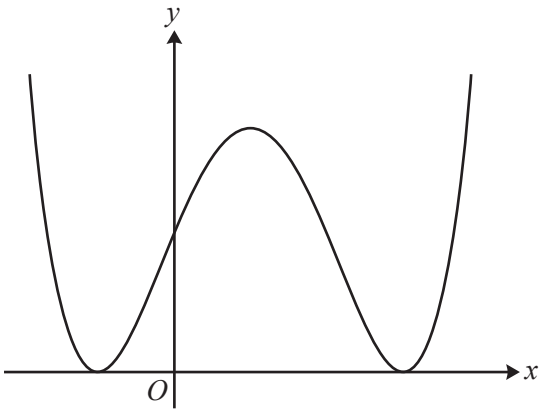
(ii) Find the sum of the first 16 terms of this geometric progression, giving your answer correct to 3 significant figures. **[2]**

(iii) Use an algebraic method to find the smallest value of N such that $\sum_{n=1}^{N} u_n > \sum_{n=1}^{\infty} w_n$. **[6]**

7 The cubic polynomial f(x) is defined by f(x) = $x^3 - 3x^2 - x + 3$.

(i) Find the quotient and remainder when f(x) is divided by ($x + 1$). **[3]**

(ii) Hence find the three roots of the equation f(x) = 0. **[3]**



The diagram shows the curve C with equation $y = x^4 - 4x^3 - 2x^2 + 12x + 9$.

(iii) Show that the x-coordinates of the stationary points on C are given by $x^3 - 3x^2 - x + 3 = 0$. **[2]**

(iv) Use integration to find the exact area of the region enclosed by C and the x-axis. **[4]**

8 **(i)** The curve $y = 3^x$ can be transformed to the curve $y = 3^{x-2}$ by a translation. Give details of the translation. **[2]**

(ii) Alternatively, the curve $y = 3^x$ can be transformed to the curve $y = 3^{x-2}$ by a stretch. Give details of the stretch. **[2]**

(iii) Sketch the curve $y = 3^{x-2}$, stating the coordinates of any points of intersection with the axes. **[2]**

(iv) The point P on the curve $y = 3^{x-2}$ has y-coordinate equal to 180. Use logarithms to find the x-coordinate of P, correct to 3 significant figures. **[3]**

(v) Use the trapezium rule, with 2 strips each of width 1.5, to find an estimate for $\int_1^4 3^{x-2}\,\mathrm{d}x$. Give your answer correct to 3 significant figures. **[3]**

9 A curve has equation $y = \sin(ax)$, where a is a positive constant and x is in radians.

(i) State the period of $y = \sin(ax)$, giving your answer in an exact form in terms of a. **[1]**

(ii) Given that $x = \frac{1}{5}\pi$ and $x = \frac{2}{5}\pi$ are the two smallest positive solutions of $\sin(ax) = k$, where k is a positive constant, find the values of a and k. **[3]**

(iii) Given instead that $\sin(ax) = \sqrt{3}\cos(ax)$, find the two smallest positive solutions for x, giving your answers in an exact form in terms of a. **[4]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1	(i)		$\frac{1}{2}$ x 8 x AB x sin30 = 20 AB = 10	M1 A1 **[2]**	Equate correct attempt at area of triangle to 20 Obtain 10	Must be using correct formula, including $\frac{1}{2}$ Allow if subsequently evaluated in radian mode (gives – 3.95AB = 20) If using $\frac{1}{2}$ x b x h then must be valid use of trig to find h Must be exactly 10
	(ii)		$BC^2 = 8^2 + 10^2 – 2$ x 8 x 10 x cos30 BC = 5.04	M1 A1 **[2]**	Attempt to use correct cosine rule, using their AB Obtain 5.04, or better	Must be using correct cosine rule Allow M1 if not square rooted, as long as BC^2 soi Allow if subsequently evaluated in radian mode (gives 11.8), but 11.8 by itself cannot imply M1 Allow if correct formula seen but is then evaluated incorrectly (using $(8^2 + 10^2 – 2$ x 8 x 10) x cos30 gives 1.86) Allow any equiv method as long as valid use of trig If > 3sf, allow answer rounding to 5.043 with no errors seen

Question			Answer	Marks	Guidance	
2	(i)		54^o x $\frac{\pi}{180} = \frac{3\pi}{10}$	M1	Attempt to use conversion factor of $\frac{\pi}{180}$	Must use $\frac{\pi}{180}$ or $\frac{2\pi}{360}$ or equiv method such as fractions of a circle Can also use 1 rad = 57.3^0 or $1^0 = 0.0175$ rad Must use fractions correct way up so multiplying by $\frac{180}{\pi}$ is M0 0.942 (or better) with no working will imply M1
				A1	Obtain $\frac{3\pi}{10}$	Allow exact simplified equiv ie 0.3π A0 if not fully simplified No ISW if decimal equiv (0.942) given as final answer However, if both decimal and exact answers seen, then allow A1 if, and only if, the exact answer is indicated as their only intended final answer (eg underlined)
				[2]		
	(ii)		$\frac{3\pi}{10} r + 2r = 60$ $r = 20.4$	M1*	Attempt perimeter in terms of r	Must be using $r\theta$ as arc length, and also including $2r$ in the perimeter attempt Allow use of an incorrect θ from **(i)** Only allow incorrect θ if seen in **(i)**, so $0.3r + 2r$ is M0, unless 0.3 was their **(i)** Could be using decimal equiv for θ (0.942) M0 if using 54^0, unless part of a valid attempt such as fractions of a circle M0 if using radians incorrectly eg 0.942π
				M1d*	Equate to 60, and attempt to solve	Must be a valid solution attempt, and go as far as an attempt at r M0 for $2.3\pi r = 60$, or similar Could be working exactly or in decimals
				A1	Obtain 20.4, or better	If > 3sf, allow answers in the range [20.39, 20.40]
				[3]		

Question			Answer	Marks	Guidance	
3	(i)		$3^3 + (3 \times 3^2 \times kx) + (3 \times 3 \times (kx)^2) + (kx)^3$ $= 27 + 27kx + 9k^2x^2 + k^3x^3$	M1	Attempt expansion	Must attempt at least 3 of the 4 terms Each term must be an attempt at the product of the relevant binomial coeff soi, the correct power of 3 and the correct power of kx Allow M1 if powers used incorrectly with kx ie only applied to the x and not to k as well Binomial coeff must be numerical, so ${}^3C_2$ is M0 until evaluated Allow M1 for expanding $c(1 + \frac{kx}{3})^3$, any c Allow M1 for reasonable attempt to expand brackets
				A1	Obtain at least two correct terms	Allow 3^3 for 27 and 3^2 for 9 Allow $(kx)^2$ and/or $(kx)^3$ unless later incorrect Terms could just be listed
				A1	Obtain at least one further correct term	Allow 3^3 for 27 and 3^2 for 9 Allow $(kx)^2$ and/or $(kx)^3$ unless later incorrect Terms could just be listed
				A1 **[4]**	Obtain fully correct simplified expansion	Must now be 27 and 9, not still index notation Allow $(kx)^2$ and/or $(kx)^3$ unless later incorrect Must be a correct expansion, with terms linked by '+' rather than just a list of 4 terms No ISW if correct final answer is subsequently spoiled by attempt to 'simplify' eg dividing by 27

Question	Answer	Marks	Guidance	
(ii)	$9k^2 = 27$ $k^2 = 3$ $k = \pm\sqrt{3}$	M1	Equate their coeff of x^2 to their constant term and attempt to solve for k	Must be equating coefficients not terms - allow recovery if next line is $k^2 = 3$, but M0 if x^2 still present at this stage Must attempt k, but allow if only positive square root is considered If a division attempt was made in part (i) then allow M1 for using either their original terms or their 'simplified' terms
		A1	Obtain $k = \pm\sqrt{3}$	Must have ±, or two roots listed separately Final answer must be given in exact form A0 for $\pm\sqrt{(27/9)}$ Must come from correct coefficients only, not from terms that were a result of a division attempt **SR** allow **B1** if $k = \pm\sqrt{3}$ is given as final answer, but inconsistent use of terms / coefficients within solution
		[2]		

Question			Answer	Marks	Guidance	
4	(i)		$\log_3 x^2 - \log_3(x+4)$ $= \log_3 \frac{x^2}{x+4}$	B1* B1d* **[2]**	Obtain $\log_3 x^2 - \log_3(x+4)$ Obtain $\log_3 \frac{x^2}{x+4}$ or equiv single term	Allow no base Could be implied if both log steps done together Allow equiv eg $2(\log_3 x - \log_3(x+4)^{0.5})$ CWO so B0 if eg $\frac{\log x^2}{\log(x+4)}$ seen in solution No ISW if subsequently incorrectly 'simplified' eg $\log_3(\frac{x}{4})$ Must now have correct base in final answer - condone if omitted earlier
	(ii)		$\frac{x^2}{x+4} = 3^2$ $x^2 = 9(x+4)$ $x^2 - 9x - 36 = 0$ $(x-12)(x+3) = 0$ $x = 12$	M1* A1 M1d* A1 **[4]**	Attempt correct method to remove logs Obtain any correct equation Attempt complete method to solve for x Obtain $x = 12$ as only solution	Equation must be of format $\log_3 f(x) = 2$, with $f(x)$ being the result of a legitimate attempt to combine logs (but condone errors such as incorrect simplification of fraction) Allow use of their **(i)** only if it satisfies the above criteria, so $x^2 - (x+4) = 9$ is M0 whether or not in **(i)** Not involving logs Solving a 3 term quadratic - see additional guidance Must attempt at least one value of x Must be from a correct solution of a correct quadratic, and A0 if other root (if given) is not $x = -3$ A0 if $x = -3$ still present Not necessary to consider $x = -3$, and then discard, but A0 if discarded for incorrect reason **NB** Despite not being 'hence' allow full credit for other valid attempts, such as combining $\log_3(x+4)$ with $\log_3 9$ on right-hand side before removing logs, or starting with $\log_3 x - \frac{1}{2}\log_3(x+4) = 1$ **SR** in **(i)** $\frac{\log x^2}{\log(x+4)}$ becoming $\log_3 \frac{x^2}{x+4}$ was penalised as an error in notation, but is eligible for full credit in **(ii)**

Question			Answer	Marks	Guidance	
5	(a)		$\int (2x^3 - 3x^2 + 4x - 6)\mathrm{d}x$ $= \frac{1}{2}x^4 - x^3 + 2x^2 - 6x + c$	M1 A1FT A1 **[3]**	Expand brackets and attempt integration Obtain at least three correct (algebraic) terms Obtain fully correct expression, including $+c$	Must be reasonable attempt to expand brackets, resulting in at least 3 terms, but allow slip(s) Integration attempt must have an increase in power by 1 for at least 3 of their terms Following their expansion Allow unsimplified coefficients Coefficients must now be fully simplified A0 if integral sign or dx still present in final answer, but allow $\int$ = …
	(b)	(i)	$\left[-6x^{-1} + 2x^{-2}\right]_1^a$ $= (-6a^{-1} + 2a^{-2}) - (-6 + 2)$ $= 4 - 6a^{-1} + 2a^{-2}$	M1 A1 M1 A1 **[4]**	Attempt integration Obtain fully correct expression Attempt correct use of limits Obtain $4 - 6a^{-1} + 2a^{-2}$ aef	Integral must be of the form $k_1x^{-1} + k_2x^{-2}$, any k_1 and k_2 as long as numerical Allow unsimplified coefficients Allow presence of $+ c$ Must be F(a) – F(1) ie correct order and subtraction Allow F(x) to be any function with indices changed from the original, even if differentiation appears to have been attempted Coefficients should now be simplified, and constant terms combined Could use negative indices, or write as fractions A0 if $+ c$ present in final answer A0 if integral sign or dx still present in final answer, but condone presence for first 3 marks ISW any subsequent work, such as further attempts at simplification, multiplying by a^2, equating to a constant, or writing as an inequality

Question			Answer	Marks	Guidance	
		(ii)	4	B1FT **[1]**	State 4, following their **(i)**	Their **(b)(i)** must be of the form $k + k_1a^{-1} + k_2a^{-2}$, with all coefficients non-zero and numerical Do not allow 4 + 0 or equiv Must appreciate that a limit is required, so B0 for <, ≈, →, 'tends to' etc Condone confusion over use of 0 and ∞ Final answer of 4 may result from starting again, rather than using their **(b)(i)**
6	**(i)**		$u_k = 5 + 1.5(k - 1)$ $5 + 1.5(k - 1) = 140$ $k = 91$	M1* M1d* A1 **[3]**	Attempt *n*th term of an AP, using $a = 5$ and $d = 1.5$ Equate to 140 and attempt to solve for k Obtain 91	Must be using correct formula, so M0 for $5 + 1.5k$ Allow if in terms of n not k Could attempt an *n*th term definition, giving $1.5k + 3.5$ Must be valid solution attempt, and go as far as an attempt at k Allow equiv informal methods Answer only gains full credit
	(ii)		$S_{16} = \frac{120(1-0.9^{16})}{1-0.9}$ $= 978$	M1 A1 **[2]**	Attempt to find the sum of 16 terms of GP, with $a = 120$, $r = 0.9$ Obtain 978, or better	Must be using correct formula If > 3sf, allow answer rounding to 977.6 with no errors seen Answer only, or listing and summing 16 terms, gains full credit

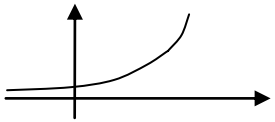
Question	Answer	Marks	Guidance	
(iii)	$\frac{1}{2}N\,(10 + (N - 1) \times 1.5) > \frac{120}{1-0.9}$ $N\,(1.5N + 8.5) > 2400$ $3N^2 + 17N - 4800 > 0$ $N = 38$	B1 B1 M1* A1 M1d* A1 **[6]**	Correct sum to infinity stated Correct S_N stated Link S_N of AP to S_∞ of GP and attempt to rearrange Obtain correct 3 term quadratic Attempt to solve quadratic Obtain $N = 38$ (must be equality)	Could be 1200 or unsimplified expression Any correct expression, including unsimplified Must be recognisable attempt at S_N of AP and S_∞ of GP, though not necessarily fully correct Allow any (in)equality sign, including < Must rearrange to a three term quadratic, not involving brackets aef - not necessary to have all algebraic terms on the same side of the (in)equation Allow any (in)equality sign See additional guidance for acceptable methods May never consider the negative root M1 could be implied by sight of 37.3, as long as from correct quadratic A0 for $N \geq 38$ or equiv in words eg 'N is at least 38' Allow A1 if 38 follows =, > or ≥ being used but A0 if 38 follows < or ≤ being used A0 if second value of N given in final answer Must be from an algebraic method - at least as far as obtaining the correct quadratic

Question			Answer	Marks	Guidance	
7	(i)		$Q = x^2 - 4x + 3$ $R = 0$	M1	Attempt complete division by $(x + 1)$, or equiv	Must be complete method to obtain at least the quotient (ie all 3 terms attempted) but can get M1A1 if remainder not considered Long division - must subtract lower line (allow one slip) Inspection - expansion must give at least three correct terms of the cubic Coefficient matching - must be valid attempt at all coeffs of the quadratic, considering all relevant terms each time Synthetic division - must be using -1 (not 1) and adding within each column (allow one slip); expect to see -1 | 1 -3 -1 3 | -1 4 (-3) | 1 -4 3 (0) The values in brackets come from attempting R and are not required for M1
				A1	Obtain fully correct quotient	Quotient could be stated explicitly, seen in division attempt or in a factorised expression for f(x). Do not ISW if their explicitly stated quotient contradicts earlier working (eg correct in division but then stated as 'quotient = 3') If using coefficient matching then $A = 1$, $B = -4$, $C = 3$ is not sufficient for A1.

Question			Answer	Marks	Guidance	
				A1 **[3]**	Obtain remainder as 0, must be stated explicitly	Not sufficient to just see 0 at bottom of division attempt (algebraic or synthetic) Allow 'no remainder' for 'remainder = 0' f(–1) = 0 is not sufficient for A1 unless identified as remainder If coefficient matching then allow $R = 0$ **SR B1** for remainder of 0 with nothing wrong seen - it could just be stated, or from f(–1), and could follow either M0 or M1 for attempt to find quotient. However, if remainder is attempted both by division attempt and f(–1) then mark final attempt at remainder
	(ii)		$x^2 - 4x + 3 = (x - 1)(x - 3)$ hence $x = -1, 1, 3$	M1 A1 B1 **[3]**	Attempt to solve their quadratic quotient Obtain $x = 1, 3$ State $x = -1$	Allow for solving any three term quadratic from their attempt at quotient, even if M0 in **(i)** See additional guidance for acceptable methods Could now be a different quotient if there is another division attempt with the factor as $(x - 1)$ or $(x - 3)$ M1A1 if both roots just stated with no method shown (but no partial credit if only one root correct) Independent of M mark B0 if $x = -1$ is clearly as result of solving their quadratic quotient only Must be seen in **(ii)** - no back credit if only seen in **(i)**
	(iii)		$\frac{dy}{dx} = 4x^3 - 12x^2 - 4x + 12$ $4x^3 - 12x^2 - 4x + 12 = 0$ hence $x^3 - 3x^2 - x + 3 = 0$ **AG**	M1 A1 **[2]**	Attempt differentiation Equate to 0 and rearrange to given answer	Decrease in power by 1 for at least 3 of the terms (could include $9 \rightarrow 0$) Not sufficient to substitute their roots to show $y = 0$ Must equate to 0 before dividing by 4

Question	Answer	Marks	Guidance	
(iv)	$\left[\frac{1}{5}x^5 - x^4 - \frac{2}{3}x^3 + 6x^2 + 9x\right]_{-1}^{3}$ $= \left(\frac{153}{5}\right) - \left(-\frac{53}{15}\right)$ $= \frac{512}{15}$	M1* A1 M1d* A1 **[4]**	Attempt integration Obtain fully correct expression Attempt correct use of correct limits Obtain $^{512}/_{15}$, or any exact equiv	Increase in power by 1 for at least 3 of the terms Must be integrating equation of curve, not f(x) Allow unsimplified coefficients Allow presence of + c No follow-through from incorrect roots in **(ii)** Must be F(3) – F(–1) ie correct order and subtraction Could find area between 1 and 3, but must double this for M1 If final area is incorrect then must see evidence of use of limits to award M1; if all that is shown is the difference of two numerical values then both must be correct eg just $\left(\frac{153}{5}\right) - \left(-\frac{23}{15}\right) = \frac{482}{15}$ is M0 as no evidence for second term Decimal equiv must be exact ie $34.1\dot{3}$, so A0 for 34.13, 34.133... etc Allow A1 if exact value seen, but followed by decimal equiv Answer only is 0/4 - need to see evidence of integration, but use of limits does not need to be explicit

Question			Answer	Marks	Guidance	
8	(i)		2 (units) in the positive *x*-direction	M1	Correct direction	Identify that the translation is in the *x*-direction (either positive or negative, so M1 for eg '2 in negative *x*-direction') Allow any terminology as long as intention is clear, such as in/on/along the *x*-axis Ignore the magnitude
				A1	Fully correct description	Must have correct magnitude and correct direction, using precise language - such as 'in the *x*-direction', 'parallel to the *x*-axis', 'horizontally' or 'to the right' A0 for in/on/along the *x*-axis etc Allow M1A1 for '2 in the *x*-direction' as positive is implied A0 for 'factor 2' 'Units' is not required, but A0 for 'places', 'spaces', 'squares' etc Allow in vector notation as well, so M1 for $\begin{pmatrix} k \\ 0 \end{pmatrix}$ and M1A1 for $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$
				[2]		
	(ii)		sf $\frac{1}{9}$ in the *y*-direction	M1	Correct direction, with sf of $\frac{1}{9}$ or 9	Identify that the stretch is in the *y*-direction, with a scale factor of either $\frac{1}{9}$ or 9 (or equiv in index notation) Allow just $\frac{1}{9}$ or 9, with no mention of 'scale factor' Allow exact decimal equiv for $\frac{1}{9}$ Allow any terminology as long as the intention is clear, such as in/on/along the *y*-axis
				A1	Fully correct description	Must have correct scale factor and correct direction, using precise language - such as 'in the *y*-direction', 'parallel to the *y*-axis' or 'vertically' A0 for in/on/along the *y*-axis etc Must now have 'scale factor' or 'factor' Allow 'positive *y*-direction' (not incorrect as graph is wholly above *x*-axis)
				[2]		

Question	Answer	Marks	Guidance	
(iii)	 intersect at $(0, \frac{1}{9})$	B1*	Correct sketch, in both quadrants	Curve must tend towards the negative x-axis, but not touch or cross it, nor a significant flick back upwards If from plotted points then there must be enough of the graph shown to demonstrate the correct general shape, including the negative x-axis being an asymptote Ignore any numerical values given
		B1d*	State $(0, \frac{1}{9})$	Condone $x = 0$, $y = \frac{1}{9}$ as an alternative, but $x = 0$ must be stated explicitly rather than implied Allow no brackets around the coordinates Allow exact decimal equiv for $\frac{1}{9}$ Allow just $\frac{1}{9}$ as long as marked on the y-axis Allow BOD for $(\frac{1}{9}, 0)$ on y-axis, but not if just stated Just being seen in a table of values is not sufficient Ignore any other labelled coordinates
		[2]		
(iv)	$\log 3^{x-2} = \log 180$ (or $x - 2 = \log_3 180$) $(x - 2)\log 3 = \log 180$ $x - 2 = 4.7268...$ $x = 6.73$	M1*	Introduce logs and drop power	Can use logs to any base, as long as consistent on both sides, and allow no explicit base as well The power must also be dropped for the M1 Brackets must be seen around the $(x - 2)$, or implied by later working If taking $\log_3$ then base must be explicit
		M1d*	Attempt to solve for x	Correct order of operations, and correct operations so M0 for $\log_3 180 - 2$ M0 if logs used incorrectly eg $x - 2 = \log(\frac{180}{3})$
		A1	Obtain 6.73, or better	If > 3sf, allow answer rounding to 6.727 with no errors seen 0/3 for answer only or T&I If rewriting eqn as $3^{x-2} = 3^{4.73}$ then 0/3 unless evidence of use of logs to find the index of 4.73 **SR** If using index rules first then **B1** for $3^x = 1620$ **M1** for attempting to use logs to solve $3^x = k$ **A1** for 6.73
		[3]		

Question	Answer	Marks	Guidance	
(v)	$0.5\times1.5\times\left\{3^{-1}+2\times3^{0.5}+3^{2}\right\}$ = 9.60	B1	State the 3 correct *y*-values, and no others	B0 if other *y*-values also found (unless not used) Allow for unsimplified, even if subsequent error made Allow decimal equivs
		M1	Attempt use of correct trapezium rule to attempt area between $x = 1$ and $x = 4$	Correct placing of *y*-values required *y*-values may not necessarily be correct, but must be from attempt at using correct *x*-values The 'big brackets' must be seen, or implied by later working Could be implied by stating general rule in terms of y_0 etc, as long as these have been attempted elsewhere and clearly labelled Could use other than 2 strips as long as of equal width (but M0 for just one strip) Must have *h* as 1.5, or a value consistent with the number of strips used if not 2
		A1	Obtain 9.60, or better (allow 9.6)	Allow answers in the range [9.595, 9.600] if > 3sf Answer only is 0/3 Using the trap. rule on the result of an integration attempt is 0/3, even if integration is not explicit Using two separate trapezia can get full marks Using other than 2 trapezia (but not just 1) can get M1 only
		[3]		

Question			Answer	Marks	Guidance	
9	(i)		$\frac{2\pi}{a}$	B1 **[1]**	State $\frac{2\pi}{a}$	Any exact equiv Allow in degrees ie $\frac{360}{a}$ B0 if given as a range eg $0 \leq x \leq \frac{2\pi}{a}$
	(ii)		$\frac{1}{5}\pi a = \pi - \frac{2}{5}\pi a$ hence $a = \frac{5}{3}$ $k = \frac{1}{2}\sqrt{3}$	M1 A1 A1 **[3]**	Attempt to use symmetry of sine curve, or equiv Obtain $a = \frac{5}{3}$ Obtain $k = \frac{1}{2}\sqrt{3}$	Allow any correct relationship between the two solutions, in radians or degrees Could also identify that the period must be $\frac{6}{5}\pi$ Any exact equiv CWO, but allow working in degrees Any exact equiv, but not involving sin CWO, but allow working in degrees A0 if from incorrect a
			Alternative solution $\sin(\frac{1}{5}\pi a) = \sin(\frac{2}{5}\pi a)$ $\sin(\frac{1}{5}\pi a) = 2\sin(\frac{1}{5}\pi a)\cos(\frac{1}{5}\pi a)$ $2\cos(\frac{1}{5}\pi a) = 1$, hence $\frac{1}{5}\pi a = \frac{\pi}{3}$ $a = \frac{5}{3}$ $k = \frac{1}{2}\sqrt{3}$	M1 A1 A1	Attempt to use correct sin2A identity Obtain $a = \frac{5}{3}$ Obtain $k = \frac{1}{2}\sqrt{3}$	As far as $2\cos(\frac{1}{5}\pi a) = 1$

Question	Answer	Marks	Guidance	
(iii)	$\tan(ax) = \sqrt{3}$ $ax = \frac{\pi}{3}, \frac{4\pi}{3}$ $x = \frac{\pi}{3a}, \frac{4\pi}{3a}$	B1	State $\tan(ax) = \sqrt{3}$	Allow B1 for correct equation even if no, or an incorrect, attempt to solve Give BOD on notation eg $\frac{\sin}{\cos}(ax)$ as long as correct equation is seen or implied at some stage Allow $\tan(ax) - \sqrt{3} = 0$, or equiv Allow B1 for identifying that $ax = \frac{\pi}{3}$ or 60^o even if equation in $\tan(ax)$ not seen – M1 would then be awarded for an attempt at x
		M1	Attempt to solve $\tan(ax) = c$	Attempt $^1/_a \tan^{-1}(c)$, any (non-zero) numerical c M0 for $\tan^{-1}(\frac{c}{a})$ Allow if attempted in degrees not radians M1 could be implied rather than explicit M1 can be awarded if using a numerical value for a
		A1	Obtain $x = \frac{\pi}{3a}$	Must be in radians not degrees Allow any exact equiv eg $\frac{\frac{\pi}{3}}{a}$ as long as intention clear - but A0 if this is then given as $\frac{a\pi}{3}$
		A1	Obtain $x = \frac{4\pi}{3a}$	Must be in radians not degrees Allow any exact equiv eg $\frac{\frac{4\pi}{3}}{a}$ as long as intention clear - but A0 if this is then given as $\frac{4a\pi}{3}$ Allow $\frac{\pi}{3a} + \frac{\pi}{a}$, unless then incorrectly simplified If more than two solutions given, then mark the two smallest ones and ISW the rest eg $\frac{\pi}{3a}, \frac{4\pi}{3a}, \frac{7\pi}{3a}$ would be A1A1 but $\frac{\pi}{3a}, \frac{2\pi}{3a}, \frac{4\pi}{3a}$ would be A1A0
		[4]		

Question	Answer	Marks	Guidance
	Alternative solution		
	$\sin^2(ax) = 3\cos^2(ax)$ $4\sin^2(ax) = 3$ or $4\cos^2(ax) = 1$ $\sin(ax) = \pm\frac{1}{2}\sqrt{3}$ or $\cos(ax) = \pm\frac{1}{2}$	**B1**	Obtain $4\sin^2(ax) = 3$ or $4\cos^2(ax) = 1$	Any correct, simplified, equation in a single trig ratio
	$ax = \frac{\pi}{3}, \frac{4\pi}{3}$ $x = \frac{\pi}{3a}, \frac{4\pi}{3a}$	**M1**	Attempt to solve $\sin^2(ax) = c$ or $\cos^2(ax) = c$	Allow M1 if just the positive square root used Attempt $\frac{1}{a}\sin^{-1}(\sqrt{c})$ or $\frac{1}{a}\cos^{-1}(\sqrt{c})$, any (non-zero) numerical c M0 for $\sin^{-1}(\frac{\sqrt{c}}{a})$ M0 for $\cos^{-1}(\frac{\sqrt{c}}{a})$ Allow if attempted in degrees not radians M1 could be implied rather than explicit M1 can be awarded if using a numerical value for a
		A1	Obtain $x = \frac{\pi}{3a}$	Must be in radians not degrees Allow any exact equiv eg $\frac{\frac{\pi}{3}}{a}$ as long as intention clear - but A0 if this is then given as $\frac{a\pi}{3}$
		A1	Obtain $x = \frac{4\pi}{3a}$	Must be in radians not degrees Allow any exact equiv eg $\frac{\frac{4\pi}{3}}{a}$ as long as intention clear - but A0 if this is then given as $\frac{4a\pi}{3}$ Allow a correct answer still in two terms, unless then incorrectly simplified If more than two solutions given, then mark the two smallest ones and ISW the rest eg $\frac{\pi}{3a}, \frac{4\pi}{3a}, \frac{7\pi}{3a}$ would be A1A1 but $\frac{\pi}{3a}, \frac{2\pi}{3a}, \frac{4\pi}{3a}$ would be A1A0